

The Normal Distribution

Chapter 6

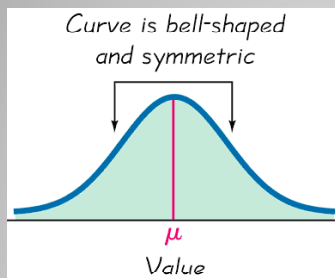
- 6-1 Introduction
- 6-2 Properties of a Normal Distribution
- 6-3 The Standard Normal Distribution
- 6-4 Applications of the Normal Distribution
- 6-5 The Central Limit Theorem
- 6-7 Summary

Outline

- Objectives:
 - Identify the properties of a normal distribution

Section 6-2 Properties of a Normal Distribution

- A normal distribution is a continuous, symmetric, bell-shaped distribution of a variable



$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

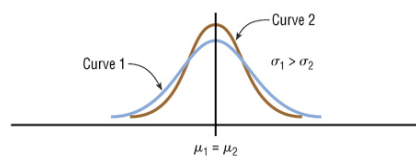
where $e \approx 2.718$

$\pi \approx 3.14$

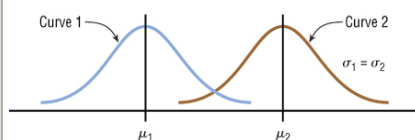
What is a Normal Distribution?

- Any particular normal distribution is determined by two parameters
 - Mean, μ
 - Standard Deviation, σ

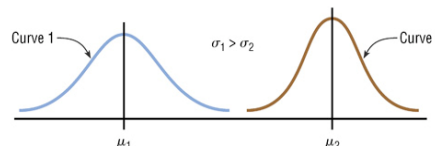
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(a) Same means but different standard deviations



(b) Different means but same standard deviations



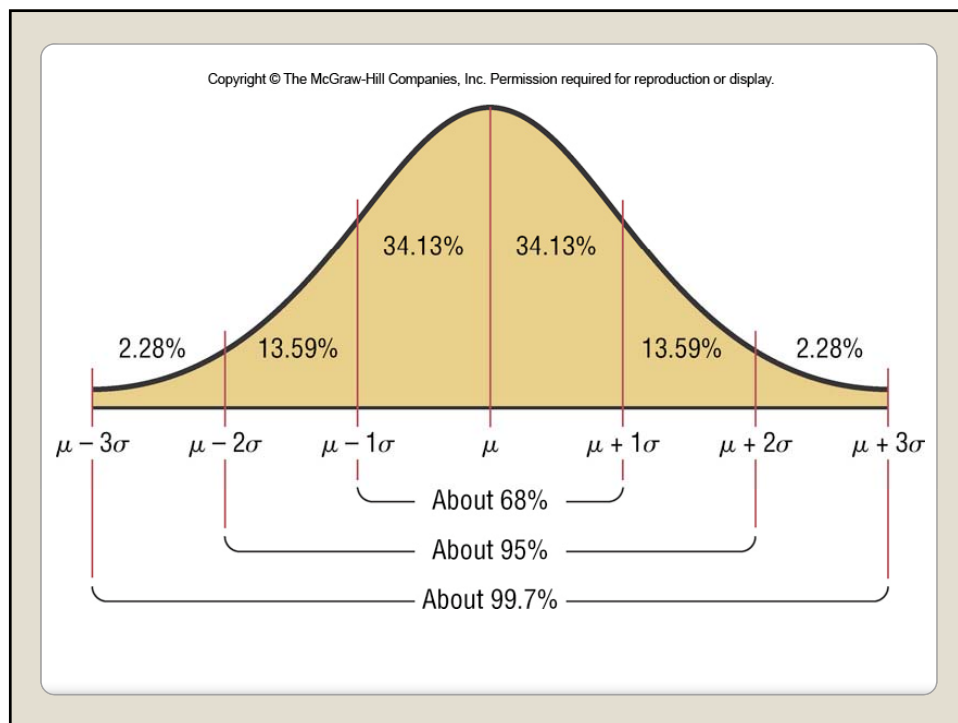
(c) Different means and different standard deviations

- A normal distribution is bell-shaped (symmetric)
- The mean, median, and mode are equal and are located at the center of the distribution
- A normal distribution curve is unimodal (it has only one mode)
- The curve is symmetric about the mean, which is equivalent to saying that its shape is the same on both sides of a vertical line passing through the center.
- The curve is continuous, that is, there are no gaps or holes. For each value of x , there is a corresponding y -value

Properties of the Theoretical Normal Distribution

- The total area under a normal distribution is equal to 1 or 100%. This fact may seem unusual, since the curve never touches the x-axis, but one can prove it mathematically by using calculus
- The area under the part of the normal curve that lies within 1 standard deviation of the mean is approximately 0.68 or 68%, within 2 standard deviations, about 0.95 or 95%, and within 3 standard deviations, about 0.997 or 99.7%. (Empirical Rule)

Properties of the Theoretical Normal Distribution



- A continuous random variable has a uniform distribution if its values are spread evenly over the range of possibilities.
 - The graph of a uniform distribution results in a rectangular shape.
 - A uniform distribution makes it easier to see two very important properties of a normal distribution
 - The area under the graph of a probability distribution is equal to 1.
 - There is a correspondence between area and probability (relative frequency)

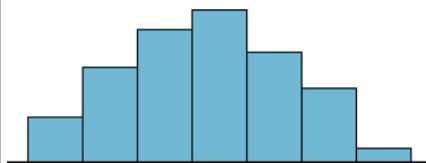
Uniform Distribution*

- Mrs. Ralston plans classes so carefully that the lengths of her classes are uniformly distributed between 50.0 minutes and 52.0 minutes (Because statistics is so interesting, they usually seem shorter). That is, any time between 50.0 min and 52.0 min is possible, and all of the possible values are equally likely. If we randomly select one of her classes and let x be the random variable representing the length of that class, then x has a uniform distribution.
 - Sketch graph
 - Find the probability that a randomly selected class will last longer than 51.5 minutes
 - Find the probability that a randomly selected class will last less than 51.5 minutes
 - Find the probability that a randomly selected class will last between 50.5 and 51.5 minutes

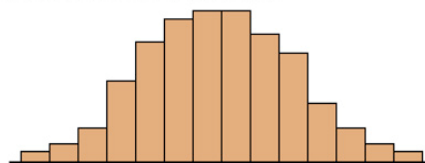
Example: Class Length

- A researcher selects a random sample of 100 adult women, measures their heights, and constructs a histogram.

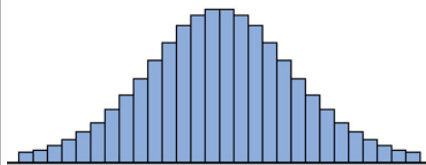
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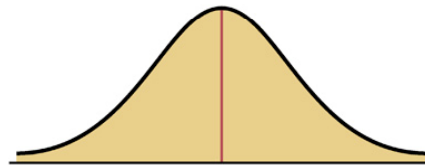
(a) Random sample of 100 women



(b) Sample size increased and class width decreased



(c) Sample size increased and class width decreased further



(d) Normal distribution for the population

- Because the total area under the normal distribution is 1, there is a correspondence between area and probability
- Since each normal distribution is determined by its own mean and standard deviation, we would have to have a table of areas for each possibility!!!!
- To simplify this situation, we use a common standard that requires only one table.

- The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

Standard Normal Distribution

- Draw a picture
- Shade the appropriate area
- Look up the appropriate z-score in Table E
- Adjust the Table E area based on your shaded picture
- While z-scores may be positive or negative (since a z-score could be above or below the mean), area is **always positive**.
- The calculated area represents the probability in the given situation.

Finding Areas Under the Standard Normal Distribution Curve

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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0										
0.1										
0.2										
⋮										
2.1										
2.2										
2.3					0.4904					
2.4										
⋮										

Table E

- Find area under the standard normal distribution curve
 - Between 0 and 1.66
 - Between 0 and -0.35
 - To the right of $z = 1.10$
 - To the left of $z = -0.48$
 - Between $z = 1.23$ and $z = 1.90$
 - Between $z = -0.96$ and $z = -0.36$
 - To left of $z = 1.31$
 - To the left of $z = -2.15$ and to the right of $z = 1.62$

Examples