Probability & Counting Rules

Chapter 4

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Chapter 4 Outline

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Section 4-1 Introduction

- Probability
  - What is probability theory?
  - How is probability computed in a variety of circumstances?
  - How is it useful to me?
    - From the time you awake until you go to bed, you make decisions regarding the possible events that are governed at least in part by chance.
      - Should I carry an umbrella today?
      - Should I accept that new job?
      - Will I have enough gas to get to school?

Probability

- In general, probability is defined as the chance (or likelihood) of an event occurring
- Probability theory is the underlying foundation on which inferential statistics is built. For example,
  - Insurance
  - Investments
  - Weather forecasting

- Objectives:
  - Determine sample spaces
  - Find the probability of an event using the
    - Classical (or theoretical) approach
    - Empirical (or experimental) approach
    - Subjective approach

Section 4-2 Sample Spaces & Probability

- **Probability experiments (aka, Chance experiment):** a chance ***process*** that leads to well-defined results
  - Flip a Coin
  - Roll a die
  - Answer multiple choice questions with 4 possibilities by guessing
- **Outcome:** the result of a single trial in a probability experiment
- **Sample Space:** the set of ***ALL*** possible outcomes of a probability experiment

Basic Concepts

- In the previous examples, the sample spaces were found by observation or reasoning, BUT what if the probability experiment is "more complex"?
  - Roll TWO dice
  - Gender of children if a family has 3 children
  - Select card from a standard 52-card deck
- If the probability experiment is "more complex", we can use a
  - Two-way Table
  - Tree Diagram: device consisting of line segments emanating from a starting point and from the outcome point

Sample Space

- **Simple Event**: an event with one outcome
  - Roll a die and a 6 shows
  - Flip a coin and a HEAD shows
- **Compound Event:** an event with two or more outcomes
  - Roll a die and an odd number shows (1,3, or 5)
  - Select a card from a deck of cards and you're interested in whether the card is red

An EVENT is a set of outcomes

Notations

- P: denotes a probability
- A, B, and C: denotes a specific event
- P(A): is read "the probability of event A"

Three Ways to Calculate Probabilities

- Classical Approach
- Empirical or Relative Frequency Probability
- Subjective Probability

Classical Probability

- First type of probability studied in 17th-18th centuries
- Assumes that all outcomes in the sample space are equally likely to occur
- P(E) (Formula—words/symbols)
- Final results can be expressed as fractions, decimals, or percentages
  - Always simply fractions
  - Round decimals to two or three places

Empirical or Relative Frequency Probability

- Relies on actual experience (experiment) to determine likelihood of outcomes
- To calculate, conduct a probability experiment and count the number of times that event E occurs, then, P(E) = (formula/symbols)

- As a probability experiment is repeated again and again, the relative frequency probability of an event tends to approach the actual probability
  - Flip coin

Law of Large Numbers

- Uses a probability value based on an educated guess or estimate, employing opinions and inexact information
  - Weather Prediction
  - Earthquake Prediction
  - Braves win pennant in 2008 Prediction

Subjective Probability

4 Basic Probability Rules

- Probability of any event E is a number (fraction or decimal) between and including 0 and 1
- $0 \leq P(E) \leq 1$
- If an event E cannot occur, its probability is 0
- P(impossible event) =0

4 Basic Probability Rules

- If event E is certain to occur, then the probability is 1.
- P(definitely happening event) = 1
- The sum of the probabilities of all the outcomes in the sample space is 1.