- Counting rules are useful in determining the number of possible outcomes for a given event

- Types of Counting Rules
  - Fundamental Counting Rule
  - Permutation Rule
  - Combination Rule

Section 4.5-Counting Rules

Fundamental Counting Rule

For a sequence of events in which the first event can occur k_1 ways and the second event can occur k_2 ways, etc. the events together can occur a total of $k_1 \bullet k_2 \bullet k_3 \ldots k_n$ ways.

Notation

- The factorial symbol **!** denotes the product of decreasing positive whole numbers.
- $n! = n\,(n-1)\,(n-2) \cdot \cdot \cdot (3)\,(2)\,(1)$
- Special Definition: $0! = 1$
- Find the **!** key on your calculator

Factorial Rule

A collection of *n* different items can be arranged or ordered in *n*! different ways.

Permutations Rule

(assuming all items are different)

n **is the number of available items**

r **is the number of items to be selected**

Then, the number of permutations (or sequences) is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Note that order is important

Combinations Rule

n **different items**

r **items to be selected**

The number of combinations is

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

Note that order is NOT important

When different orderings of the same items are to be counted separately (as a separate outcome), we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.

- Suppose that you flip a coin a coin and roll a die. Determine the number of possible outcomes to the experiment.
- Using the fundamental counting rule:

$$2 \bullet 6 = 12$$

Example

- Suppose that you decide to choose 3 students from our class of 21 people to attend a statistics conference. How many possible sets of attendees are possible?

$$_nC_r = {}_{21}C_3 = \frac{21!}{3!(21-3)!} = 1330$$

Example

- Suppose that you decide to award 2 customers a special gift card for visiting your store. One card is for $1000 and the other one is for $100. If you have 30 customers on a particular day, how many ways could you award the cards?

$$_nP_r = {}_{30}P_2 = \frac{30!}{(30-2)!} = 870$$

Example