

## Section 4.4 Multiplication Rules & Conditional Probability

- Objectives:
  - Determine if a compound event is independent or dependent
  - Find the probability of compound events, using the multiplication rules
  - Find the conditional probability of an event
  - Find the probability of an "at least one" event

- RECALL: **Compound Event:** any event containing two or more simple events
- KEY WORD: **AND**
  - Two or more events occur in sequence
    - If a coin is tossed and then a die is rolled, you can find the probability of getting a head on the coin *and* a 4 on the die
    - If two dice are rolled, you can find the probability of getting a 6 on the first die *and* getting a 3 on the second die
  - One event displays two characteristics
    - If a card is drawn, you can determine if the Jack *and* a Diamond

**To determine the probability of a compound event involving AND, we must first determine if the two events are independent or dependent**

#### **Independent**

- Two events A and B are independent if the fact that A occurs does not affect or influence the likelihood of B occurring

#### **Dependent**

- Two events A and B are dependent if the fact that A occurs influences (or changes) the likelihood of B occurring

**Independent vs Dependent**

## Sampling with or without Replacement

- Sampling with replacement means that after sampling an item from a population, you return the item to the population. If you sample the population a second time, you could pick the same item.
- Sampling without replacement means that after choosing an item from a population, you do NOT return the item to the population.
- If you sample a population more than once without replacement, you generate a series of dependent events. If you sample with replacement, the series of events are independent.

- Drawing a card from a standard deck and getting a queen, replacing it, and drawing a second card and getting a queen
- An drawer contains 3 red socks, 2 blue socks, and 5 white socks. A sock is selected and its color is noted. The first sock is not returned to the drawer. A second sock is selected and its color noted.
- Being a lifeguard and getting a suntan
- Randomly selecting a TV viewer who is watching "The Daily Show." Randomly selecting a second TV viewer who is watching "The Daily Show."

## Examples

### Independent Events

- $P(A \text{ and } B) = P(A) \cdot P(B)$

- To find the probability of two independent events occurring simultaneously or in sequence, find the probability of each event separately and then multiply the answers.

### Dependent Events

- $P(A \text{ and } B) = P(A) \cdot P(B|A)$

- To find the probability of two dependent events occurring simultaneously or in sequence, find the probability of the first event, "adjust" the probability of the second event based on the fact that the first has occurred, and then multiply the answers.

## Multiplication Rules

- $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

- The probability of the second event B should take into account the fact that the first event A has already occurred

- KEY WORDS: **GIVEN**

- $P(B|A)$  is read as "the probability of B given A"

## Conditional Probability

- Draw two cards with replacement from a standard deck. Find  $P(\text{Queen and Queen})$
- An drawer contains 3 red socks, 2 blue socks, and 5 white socks. Two socks are selected without replacement. Find  $P(\text{White and Red})$
- A new computer owner creates a password consisting of two characters. She randomly selects a letter of the alphabet for the first character and a digit (0-9) for the second character. What is the probability that her password is "K9"? Would this password be an effective deterrent to a hacker?
- In the 108<sup>th</sup> Congress, the Senate consisted of 51 Republicans, 48 Democrats, and 1 Independent. If a lobbyist for the tobacco industry randomly selects three different Senators, what is the probability that they are all Republicans? That is  $P(\text{Republican and Republican and Republican})$ .

## Examples

	Republican	Democrat	Independent
Male	46	39	1
Female	5	9	0

If we randomly select one Senator, find the probability

$P(\text{Republican given that a male is selected})$

$P(\text{Male given that a Republican is selected})$

$P(\text{Female given that an Independent was selected})$

$P(\text{Democrat or Independent given that a male is selected})$

- "At least one" is equivalent to "one or more"
- The **complement** (not) of getting at least one item of a particular type is that you get NO items of that type
- To calculate the probability of "at least one" of something, calculate the probability of NONE, then subtract the result from 1.

$$P(\text{at least one}) = 1 - P(\text{none})$$

### "At Least One" Probability

- If a couple plans to have 4 children, what is the probability that there will be at least one girl?
- In acceptance sampling, a sample of items is randomly selected without replacement and the entire batch is rejected if there is at least one defect. The Medtyme Company just manufactured 5000 blood pressure monitors and 4% are defective. If 3 of the monitors are selected at random and tested, what is the probability that the entire batch will be rejected?

### Examples