9.1 RATIONAL EXPONENTS

Roots and Rational Expressions

- Radical expression – an expression written with a radical or a root (power)
- $\sqrt[n]{\ }$ is called a radical
- Radicand is the expression under the radical
- n is the index of the root, if blank n=2
- The nth root of x , $\sqrt[n]{x} = x$

where $x \geq 0$ if n is even (can't take a root of a negative number if the index is even)

Examples

1. $\sqrt{4} = \pm 2$
2. $\sqrt{-4}$ = not a real # since there is no real number whose square is -4
3. $\sqrt[3]{27} = 3$
4. $\sqrt[3]{-27} = -3$ since $(-3)^3 = -27$
5. $-\sqrt[4]{16} = -2$
6. $\sqrt[4]{-16}$ = no real # since $x^4 \neq 16$

Note:

- If there is a variable under the radical we will assume it represents nonnegative numbers since we can't take an even root of a negative number
- Ex:

$\sqrt[4]{(7b)^4} = 7b$ (b assumed positive)

correct notation:

$\sqrt[4]{(7b)^4} = 7|b|$ (makes it positive)

Rational Exponents

$$\sqrt[n]{x} = x^{1/n} \qquad \text{for } x \geq 0$$

(root) (rational exponent)

Ex: Rewrite without rational exponents (as a root)

1. $y^{1/7}$
2. $16^{1/2}$
3. $(64/125)^{1/3}$

Rational Exponent Theorem

- $x^{m/n} = (x^{1/n})^m = (x^m)^{1/n}$
- $x^{m/n}$ means $(\sqrt[n]{x})^m$ or $\sqrt[n]{x^m}$

Hint: Eliminate () and negative exponents first

- Ex:

1. $81^{3/4}$
2. $(16/49)^{-1/2}$
3. $(a^{1/3}b^4)/(a^{3/5}b^{1/3})$
4. $(r^{-5}s^{1/2})^4/(r^{12}s^{5/2})$