

Section 6.5 Application Problems Helps and Hints

- **Number Problems**

The key here is to choose a baseline variable to represent a number that all other numbers are compared to. These types of problems require a little more thought because you may have a slightly different scenario each time. After choosing a variable, translate comparisons into expressions for the other numbers. Enough information will be given to write an equation that can be solved for the unknown variable.

Example 1:

The sum of the reciprocals of two consecutive integers is $7/12$. Find the two integers.

Let $x = 1^{\text{st}}$ integer. Then the 2^{nd} integer = $x + 1$

The reciprocal of x is $1/x$. The reciprocal of $x + 1$ is $\frac{1}{x+1}$.

$$\frac{1}{x} + \frac{1}{x+1} = \frac{7}{12}$$

$$x = \frac{-4}{7} \text{ or } 3$$

Since $-4/7$ is not an integer, the only solution possible is 3. Thus, the two integers are 3 and 4.

Example 2:

The numerator of a certain fraction is 2 more than the denominator. If $1/3$ is added to the fraction, the result is 2. Find the fraction.

Let $x = \text{numerator}$ and $x = 2 + \text{denominator}$, then denominator = $x - 2$.

$$\frac{1}{3} + \frac{x}{x-2} = 2$$

$$x = 5$$

The fraction is $5/3$.

- **Motion Problems**

Recall: distance = rate X time

By rearranging the terms, two equivalent forms of this equation are

rate = distance \div time and time = distance \div rate

The following table will be helpful in solving motion problems where A and B are the different legs of the trip.

	D	R	T
A			
B			

Use the appropriate space to fill in the known information. If time is the same you can use an alternative form of the distance-rate formula, $t = d/r$, and set the two resulting expressions equal to one another to solve for the unknown.

Example 1: Time is equal

An airplane flying against the wind travels 500 miles in the same amount of time that it would take to travel 600 miles with the wind. If the speed of the wind is 50 miles per hour, what is the speed of the plane in still air?

	D	R	T
Against	500	$r - 50$	t
With	600	$r + 50$	t

Where R is the rate of the plane in still air.

$$t = \frac{d}{r} \quad t_a = t_w \quad t_a = \frac{500}{r-50} \quad t_w = \frac{600}{r+50}$$

$$\frac{500}{r-50} = \frac{600}{r+50}$$

$$r = 550 \text{ mph}$$

The rate of the plane in still air is 550mph.

Sometimes information will have to be inferred from other information that you do know. For example, if only the total time is known you are going to have to use the distance and rate information to find the expression for the times of A and B. You can then use $t_A + t_B = \text{total time}$ and substitute in each of the time expressions and solve accordingly.

Example 2: Total time only

To train for a bicycle portion of the race, Jerri rides 24 miles out a straight road, then turns around and rides 24 miles back. The trip out is against the wind, whereas the trip back is with the wind. If she rides 10 miles per hour faster with the wind than she does against the wind, and the complete trip out and back takes 2 hours, how fast does she ride when she rides against the wind?

	D	R	T
Against	24	r	t_a
With	24	$r + 10$	t_w

Where R is the Jerri's rate against the wind.

In this example, the total time of the ride is 2 hours which can be represented using the formula

$t_a + t_w = 2$. We can find the expressions for t_a and t_w by using $t = \frac{d}{r}$ where we know d and r.

$$t_a = \frac{24}{r} \text{ and } t_w = \frac{24}{r + 10}$$

Substituting them into $t_a + t_w = 2$ to solve for the unknown variable r.

$$\frac{24}{r} + \frac{24}{r + 10} = 2$$

$$r = 20 \text{ or } -6$$

You can't have a negative rate so -6 is not a solution. Jerri's rate against the wind is 20mph.

- **Work Problems**

Using the following system you can solve most work problems with ease.

Let a = time it takes A to do the job alone

Let b = time it takes B to do the job alone

Let t = the time to do the job together

$$\text{Then } \frac{t}{a} + \frac{t}{b} = 1$$

The trick here is to read the problem carefully enough to determine if A and B are working together or against one another. If A and B are working together then you will be adding while you will be subtracting if they are working against one another. The total job will always be equal to 1 because that represents the job being 100% complete. The fractional time of A plus or minus the fractional time of B equals the complete job.

Example 1 : Working together

Matt can paint the room in 3 hours while it takes Todd 6 hours to paint the same room. How long would it take to paint the room if both Matt and Todd worked together?

Let t = time to paint the room together

Matt's time + Todd's time = completely painted

$$\frac{t}{3} + \frac{t}{6} = 1$$

$$2t + t = 6$$

$$3t = 6$$

$$t = 2$$

Together they can paint the room in 2 hours.

Example 2: Working Against One Another

A water tank can be filled in 20 hours while it takes 25 hours to drain it. How long will it take to fill the tank if both pipes are left open?

Let t = time to fill the tank together

Time to fill – time to empty = completely filled

$$\frac{t}{20} - \frac{t}{25} = 1$$

$$5t - 4t = 100$$

$$t = 100$$

Together they can fill the tank in 100 hours.

Note: If they were working together to empty the tank the equation would have been written

Time to empty – time to fill = completely empty

$$\frac{t}{25} - \frac{t}{20} = 1$$

Which in this case the tank never would be able to empty.